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Minimum distance bounds for linear codes over GF(11)

Research Article

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Abstract: Let $[n, k, d]_q$ code be a linear code of length n, dimension k and minimum Hamming distance d over $GF(q)$. One of the most important problems in coding theory is to construct codes with best possible minimum distances. In this paper 36 new cyclic and quasi-cyclic (QC) codes over GF(11) are presented and the table from [4] is enlarged by adding three new dimensions.

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1. Introduction

Let GF(q) denote the Galois field of q elements and let V(n,q) denote the vector space of all ordered n-tuples over GF(q). The Hamming weight of a vector x, denoted by $wt(x)$, is the number of nonzero entries in x. A linear code C of length n and dimension k over GF(q) is a k-dimensional subspace of V(n,q). Such a code is called $[n, k, d]_q$ code if its minimum Hamming distance is d. For linear codes, the minimum distance is equal to the smallest of the weights of the nonzero codewords.

A $k \times n$ matrix G having as rows the vectors of a basis of a linear code C is called a generator matrix for C. Let A_i denote the number of codewords of C with weight i. The weight distribution of C is the list of numbers A_i. The weight distribution $A_0 = 1$, $A_d = \alpha$, ..., $A_n = \beta$ is expressed as $0^1 d^\alpha \dots n^\beta$ also.

The orthogonal code $C^\perp$ of C is the set of words of length n that are orthogonal to all codewords in C, with respect to the ordinary inner product. The following remarkable theorem of J. MacWilliams gives a relationship between the weight distribution of a given code and its orthogonal code.

Theorem 1.1. *(The MacWilliams' identities ([17])) Let the codes C and $C^\perp$ have weight distributions $\{A_i\}$ and $\{B_i\}$ $(0 \le i \le n)$ respectively. Then:*

$$\sum_{i=0}^{n} K_t(i).A_i = q^k B_t \qquad for \quad t = 0, 1, \dots, n$$

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where

$$K_t(i) = \sum_{j=0}^{t} (-1)^j \binom{n-i}{t-j} \binom{i}{j} (q-1)^{t-j}$$

are the Krawtchouk polinomials of degree t.

A central and fundamental problem in coding theory is to find the optimal values of the parameters of a linear code. This optimization problem, known as the Main Coding Theory Problem ([16]), can be formulated in three different ways:

1: Given q, k, d, find $n_q(k,d)$, the smallest value of n for which there exists an $[n,k,d]_q$-code.

2: Given q, n, d, find $k_q(n,d)$, the largest value of k for which there exists an $[n,k,d]_q$-code.

3: Given q, n, k, find $d_q(n,k)$, the largest value of d for which there exists an $[n,k,d]_q$-code.

A code which achieves one of these three values is called an *optimal* code. All these problens are difficult but the basic two versions in the literature are 1 and 3. They have been completely solved only over small finite fields and small dimensions: $q = 2 \quad k \leq 8$; $q = 3 \quad k \leq 5$; $q = 4 \quad k \leq 4$; and $q = 5, 7, 8, 9 \quad k \leq 3$ (see [13]).

A well-known lower bound for $n_q(k,d)$ is the Griesmer bound

$$n_q(k,d) \geq g_q(k,d) = \sum_{j=0}^{k-1} \lceil \frac{d}{q^j} \rceil$$

($\lceil x \rceil$ denotes the smallest integer $\geq x$). Codes with parameters $[g_q(k,d), k, d]_q$, are called *Griesmer codes*.

By using the standard puncturing and shortening constructions, the following bounds are derived:

$$d_q(n+1,k) \leq d_q(n,k) + 1 \qquad\qquad d_q(n+1,k+1) \leq d_q(n,k) \tag{1}$$

The problem of finding the parameters of optimal codes is a very difficult one (see [12]). It has two aspects - one involves the construction of new codes with better minimum distances and the other is proving the nonexistence of codes with given parameters. For the first aspect, computer search is often used, but it is a well known fact that computing the minimum distance of a linear code is an NP-hard problem [18]. Since it is not possible to carry out exhaustive searches for linear codes with large dimension, it is natural to focus one's effort on subclasses of linear codes having rich mathematical structures. The subclass of quasi-cyclic codes is known to have such a nice structure and it has been shown in recent years that it contains many new good linear codes [3-11,14,15].

The Grassl's online database [13] is well known in the coding theory research community for the best-known linear codes (BKLC) over small fields. The best-known QC and QT codes are stored also in the Chen's online database of QT codes [2]. As new codes are discovered, these databases are updated. These two databases store codes over finite fields having sizes up to 9 and 19 respectively. Similarly, a smaller database of codes over GF(11), GF(13) and GF(17) are given in [4], [5] and [6]. MAGMA software also has a similar database included.

The remainder of the paper is organized as follows. In Section 2, some basic definitions and facts on QC codes are presented. In Section 3, thirty six new good cyclic and quasi-cyclic codes are constructed using a combinatorial computer search. In Section 4 a new table on $d_{11}(n,k)$ is presented.

2. Quasi-cyclic codes

A code C is said to be quasi-cyclic (QC or p-QC) if a cyclic shift of a codeword by p positions results in another codeword. A cyclic shift of an m-tuple $(x_0, x_1, \ldots, x_{m-1})$ is the m-tuple $(x_{m-1}, x_0, \ldots, x_{m-2})$. The blocklength, n, of a p-QC code is a multiple of p, so that $n = pm$.

A matrix B of the form

$$B = \begin{bmatrix} b_0 & b_1 & b_2 & \cdots & b_{m-2} & b_{m-1} \\ b_{m-1} & b_0 & b_1 & \cdots & b_{m-3} & b_{m-2} \\ b_{m-2} & b_{m-1} & b_0 & \cdots & b_{m-4} & b_{m-3} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ b_1 & b_2 & b_3 & \cdots & b_{m-1} & b_0 \end{bmatrix}, \tag{2}$$

is called a *circulant matrix*. A class of QC codes called 1-generator QC codes, can be constructed from $m \times m$ circulant matrices. In this case, a generator matrix, G, can be represented as

$$G = [B_1, B_2, \ldots, B_p], \tag{3}$$

where B_i is a circulant matrix.

The algebra of $m \times m$ circulant matrices over $GF(q)$ is isomorphic to the algebra of polynomials in the ring $GF(q)[x]/(x^m - 1)$ if B is mapped onto the polynomial, $b(x) = b_0 + b_1 x + b_2 x^2 + \cdots + b_{m-1} x^{m-1}$, formed from the entries in the first row of B. The $b_i(x)$'s associated with a QC code are called the *defining polynomials.*

If the defining polynomials $b_i(x)$ contain a common factor which is also a factor of $x^m - 1$ then the QC code is called *degenerate.*

The dimension k of a QC code is equal to the degree of $h(x)$, where

$$h(x) = \frac{x^m - 1}{\gcd\{x^m - 1, b_1(x), b_2(x), \cdots, b_p(x)\}}. \tag{4}$$

If the polynomial $h(x)$ has degree m, the dimension of the code is m, and (3) is a generator matrix. If $\deg(h(x)) = k < m$, a generator matrix for the code can be constructed by deleting $m - k$ rows of (3).

3. New QC codes over GF(11)

In recent years there has been an increased interest in codes over GF(11). In [15] Gulliver constructed quasi-twisted (QT) codes over GF(11) for $k \leq 7$. In [4] E. Chen and N. Aydin constructed 45 new QC and QT codes over GF(11) and presented a database for codes of dimensions $k \leq 7$ and $n \leq 150$. In [11] 34 new QC codes over GF(11) in dimensions $k = 8, 9$ are presented. Here we present many new codes in dimensions $k = 8, 9, 10$ and enlarge the database in [4] with three new columns.

The lower bounds in the table are obtained by two algorithms:

The first tool we use to establish lower bounds is the construction of QC $[pk, k, d]_{11}$ codes with large minimum distance by an iterative heuristic search algorithm. The search is initialized with an arbitrary $[pk, k, d_1]_{11}$ code ($d_1 < d$). The numbers p, k and d_1 are fixed. An upper bound U_{nd} on the number of codewords with minimum weight is also fixed (for example $U_{nd} := 99999$). The set of defining polynomials for this dimension is known (see [11]). In each iteration one of the polynomials is replaced by a new one, so that the minimum distance increases. The goal is to obtain a $[pk, k, d]_{11}$ code. When such a polynomial is found $U_{nd} := A_d - q + 1$ and the search continues with this new bound. This process continues until the specified number of iterations is exceeded. The last obtained code has the smallest number of codewords with minimum weight. It is natural to expect that such a choice will provide a greater chance of obtaining a good QC $[(p+1)k, k]_{11}$ code in the next extension. All the codes in [11] and 10 codes in Theorems 3.3 and 3.4 are constructed in this way.

The second tool we use is the well-known ASR algorithm [1], [8-10].

The upper bounds in the table follow from the upper bounds in [4] and (1).

The elements of the field GF(11) are denoted by 0, 1, 2, ... , 9, $10 = A$.

Codes with parameters

$$[24,8,14]_{11},\ [48,8,33]_{11},\ [72,8,53]_{11},\ [104,8,80]_{11},\ [112,8,87]_{11},\ [120,8,94]_{11},\ [136,8,107]_{11},$$
$$[144,8,114]_{11},\ [27,9,15]_{11},\ [72,9,51]_{11},\ [99,9,73]_{11},\ [126,9,96]_{11},\ [135,9,104]_{11},\ [153,9,119]_{11}$$

are constructed in [11]. The minimum distances of all basic codes from [11] and the codes presented in the next three theorems are in boldface type in the table for $d_{11}(n,k)$.

In [4], $d_{11}(17,7) \le 10$ and $d_{11}(16,7) \le 9$. It follows from (1) that $d_{11}(19,9) \le 10$ and $d_{11}(19,10) \le 9$. In the next theorem we present codes with parameters $[19,9,10]_{11}$ and $[19,10,9]_{11}$. So these two codes are optimal.

Theorem 3.1. *$d_{11}(19,9) = 10$ and $d_{11}(19,10) = 9$.*

Proof. There exist cyclic codes with parameters $[19,9,10]_{11}$ and $[19,10,9]_{11}$. The coefficients of the generator polynomials and weight distributions of the codes are as follows:

- A $[19,9,10]_{11}$ code:

1806766162100000000
$0^{1}\ 10^{51300}\ 11^{294120}\ 12^{1846800}\ 13^{10613400}\ 14^{44836200}\ 15^{149157600}\ 16^{373963320}\ 17^{659042550}\ 18^{732609600}\ 19^{385532800}$

- A $[19,10,9]_{11}$ code:

a821604991000000000
$0^{1}\ 9^{51300}\ 10^{410780}\ 11^{3064320}\ 12^{21557400}\ 13^{115231200}\ 14^{492309000}\ 15^{1645010880}\ 16^{4108537770}\ 17^{7252528950}\ 18^{8057732800}$
$19^{4240990200}$ □

Corollary 3.2. *1. $d_{11}(19-i, 9-i) = 10$ for $i = 1,2,3$.*
2. $d_{11}(19-i, 10-i) = 9$ for $i = 1,2,3$.
3. $d_{11}(17-i, 9-i) = 8$ for $i = 0,1$.

Theorem 3.3. *There exist cyclic codes with parameters:*

$$[24,10,12]_{11},\ [38,9,24]_{11},\ [43,8,29]_{11},\ [61,8,44]_{11},\ [61,9,43]_{11},\ [95,8,74]_{11},\ [95,10,70]_{11}.$$

Proof. The coefficients of the generator polynomials and weight distributions of the codes are as follows:

- A $[24,10,12]_{11}$ code:

A8A9A21560849510000000000

$0^{1}\ 12^{10360}\ 13^{59520}\ 14^{536640}\ 15^{3391360}\ 16^{19395150}\ 17^{91144080}\ 18^{354773160}\ 19^{1118228160}\ 20^{2799379500}\ 21^{5329227040}$
$22^{7267928760}\ 23^{6320070720}\ 24^{2633280150}$

- A $[38,9,24]_{11}$ code:

A4494468373A33901314444A94164100000000

$0^{1}\ 24^{5700}\ 25^{37620}\ 26^{186200}\ 27^{752400}\ 28^{2893320}\ 29^{10386920}\ 30^{30742380}\ 31^{79622160}\ 32^{174410880}\ 33^{315550860}\ 34^{465831930}$
$35^{531833560}\ 36^{443211670}\ 37^{239376820}\ 38^{63105270}$

- A $[43,8,29]_{11}$ code:

A53022561A5512258769738A5A24250545210000000

$0^{1}\ 29^{3440}\ 30^{6020}\ 31^{60200}\ 32^{228760}\ 33^{650160}\ 34^{2043790}\ 35^{5110980}\ 36^{11370490}\ 37^{21966980}\ 38^{34100290}\ 39^{43891820}\ 40^{43765400}$
$41^{32504990}\ 42^{15029360}\ 43^{3626200}$

The orthogonal code is a $[43,35,6]_{11}$ code with $B_6 = 27090$.

- $[61, 8, 44]_{11}$ code:

  A690A2360220A4628128966A1100AA155239A395710990589102510000000

  $0^{1}\ 44^{9150}\ 45^{8540}\ 46^{39040}\ 47^{145180}\ 48^{441030}\ 49^{1104710}\ 50^{2559560}\ 51^{5957260}\ 52^{11083090}\ 53^{18850220}\ 54^{27842840}\ 55^{35449540}\ 56^{38042040}\ 57^{33737880}\ 58^{22753000}\ 59^{11864500}\ 60^{3792980}\ 61^{678320}$

- A $[61, 9, 43]_{11}$ code:

  173788466A017A2A50776187A898A78167705A2A710A66488737100000000

  $0^{1}\ 43^{23180}\ 44^{50630}\ 45^{226920}\ 46^{584380}\ 47^{1565260}\ 48^{4040030}\ 49^{11680890}\ 50^{28932300}\ 51^{65073580}\ 52^{123759240}\ 53^{210378630}\ 54^{304248480}\ 55^{384708700}\ 56^{415491130}\ 57^{375095100}\ 58^{259332960}\ 59^{125147600}\ 60^{37637610}\ 61^{9971070}$

  The orthogonal code is a $[61, 52, 6]_{11}$ code with $B_6 = 157380$.

- A $[95, 8, 74]_{11}$ code:

  664270A9896937AA10401744354A4373561A905965399057A312503569A1A07770351370A7701630721199810000000

  $0^{1}\ 74^{34200}\ 75^{39900}\ 76^{87450}\ 77^{311600}\ 78^{599450}\ 79^{1330950}\ 80^{2764690}\ 81^{5332350}\ 82^{8800800}\ 83^{13715150}\ 84^{19373350}\ 85^{24840600}\ 86^{29663750}\ 87^{30624200}\ 88^{27928100}\ 89^{21729350}\ 90^{14448740}\ 91^{7843200}\ 92^{3414300}\ 93^{1232150}\ 94^{216600}\ 95^{28000}$

  The orthogonal code is a $[95, 87, 5]_{11}$ code with $B_5 = 18240$.

- A $[95, 10, 70]_{11}$ code:

  62769060374892705625A3946656704302A21A83559982899090551A114963581886415331902091102161000000000

  $0^{1}\ 70^{19950}\ 71^{54150}\ 72^{231800}\ 73^{557650}\ 74^{1919950}\ 75^{5232600}\ 76^{13456750}\ 77^{33813350}\ 78^{78145100}\ 79^{166665150}\ 80^{334399810}\ 81^{619286950}\ 82^{1059547350}\ 83^{1657623650}\ 84^{2362985350}\ 85^{3064592650}\ 86^{3562556050}\ 87^{3682919150}\ 88^{3353668150}\ 89^{2632053850}\ 90^{1754295460}\ 91^{966708600}\ 92^{419477250}\ 93^{135483300}\ 94^{28786900}\ 95^{2943680}$

$\square$

Theorem 3.4. *There exist 8 and 9 dimensional quasi-cyclic codes with parameters:*

$$[24, 9, 13_{11},\ [27, 8, 16]_{11},\ [32, 8, 20]_{11},\ [32, 9, 19]_{11},\ [40, 9, 25]_{11},\ [48, 9, 32_{11},\ [64.9.45]_{11},$$
$$[80, 8, 60]_{11},\ [81, 9, 59]_{11},\ [96, 9, 71]_{11},\ [114, 9, 87]_{11},\ [117, 9, 89]_{11},\ [129, 8, 102]_{11},\ [144, 9, 112]_{11}.$$

Proof. The coefficients of the defining polynomials and weight distributions of the codes are as follows:

- A $[24, 9, 13]_{11}$ code:

  A65100000000, 5557A8591000

  $0^{1}\ 13^{5520}\ 14^{50700}\ 15^{315640}\ 16^{1754610}\ 17^{8285040}\ 18^{32177320}\ 19^{101853360}\ 20^{254352360}\ 21^{484404880}\ 22^{660873660}\ 23^{574470360}\ 24^{239404240}$

- A $[27, 8, 16]_{11}$ code:

AA70A4806, AA8A72233, AA82498A5

  $0^{1}\ 16^{1710}\ 17^{14580}\ 18^{77850}\ 19^{368100}\ 20^{1441710}\ 21^{4832850}\ 22^{13211010}\ 23^{28722150}\ 24^{47770950}\ 25^{57433500}\ 26^{44130960}\ 27^{16353510}$

- A $[32, 8, 20]_{11}$ code:

AAA986A2, AAAA7022, AAAA9A68, A9076258

  $0^{1}\ 20^{2080}\ 21^{14320}\ 22^{64960}\ 23^{288320}\ 24^{1057650}\ 25^{3426080}\ 26^{9212680}\ 27^{20416480}\ 28^{36508380}\ 29^{50380400}\ 30^{50357800}\ 31^{32469920}\ 32^{10159810}$

- A $[32, 9, 19]_{11}$ code:

19A49A4100000000, 1193325330935100

  $0^{1}\ 19^{3360}\ 20^{27360}\ 21^{148960}\ 22^{706880}\ 23^{3133440}\ 24^{11782160}\ 25^{37553920}\ 26^{101150720}\ 27^{225046880}\ 28^{401363760}\ 29^{554218080}\ 30^{553663040}\ 31^{357495040}\ 32^{111654090}$

- A $[40, 9, 25]_{11}$ code:

AAAA404873, AAAAA89697, AAAA202A07, AAAA6AA846

$0^{1} 25^{2900}$ 26^{11700} 27^{64900} 28^{288250} $29^{1199900}$ $30^{4416090}$ $31^{14258800}$ $32^{40067900}$ $33^{97104800}$ $34^{199971750}$ $35^{342926560}$ $36^{476017300}$ $37^{514564400}$ $38^{406760500}$ $39^{208132700}$ $40^{52159240}$

- A $[48, 9, 32]_{11}$ code:

A65100000000, 5557A8591000, 326808475100, 213866485100

0^{1} 32^{5160} 33^{26400} 34^{122160} 35^{466200} $36^{1712650}$ $37^{5454120}$ $38^{15911220}$ $39^{40750880}$ $40^{91746120}$ $41^{178965240}$ $42^{298191720}$ $43^{416085120}$ $44^{473165010}$ $45^{420155840}$ $46^{274262340}$ $47^{116605800}$ $48^{24321710}$

- A $[64, 9, 45]_{11}$ code:

14A94A9100000000, 6723280279AA1000, 6901994221551000, A287142542611000

0^{1} 45^{4480} 46^{20000} 47^{73440} 48^{263940} 49^{848000} $50^{2537440}$ $51^{6916640}$ $52^{17347960}$ $53^{39326880}$ $54^{80240080}$ $55^{145617280}$ $56^{234101640}$ $57^{328499680}$ $58^{396701360}$ $59^{403262560}$ $60^{335945800}$ $61^{220477760}$ $62^{106640320}$ $63^{33834720}$ $64^{5287710}$

- A $[80, 8, 60]_{11}$ code:

20A1354A49110234528A34210A13414310000000, 31833AA821226616105608A83823351715510000

0^{1} 60^{3640} 61^{10400} 62^{41800} 63^{108400} 64^{283650} 65^{718960} $66^{1549600}$ $67^{3302000}$ $68^{6304200}$ $69^{10909600}$ $70^{17247560}$ $71^{24163600}$ $72^{30569300}$ $73^{33282400}$ $74^{31281000}$ $75^{25162320}$ $76^{16600400}$ $77^{8555600}$ $78^{3310200}$ 79^{843200} 80^{111050}

- A $[81, 9, 59]_{11}$ code:

AAAA98708, AAAAA7023, AAAAA9949, AA98113A9, AAA9A9A59, AAA520467, AAA249519, AA9A9A937, AA9A95490

0^{1} 59^{2790} 60^{13770} 61^{51210} 62^{164430} 63^{473800} $64^{1353870}$ $65^{3501540}$ $66^{8535630}$ $67^{19076580}$ $68^{39278160}$ $69^{73912710}$ $70^{127019340}$ $71^{196471800}$ $72^{273230790}$ $73^{336441420}$ $74^{363790710}$ $75^{339544260}$ $76^{268236000}$ $77^{174294900}$ $78^{89112270}$ $79^{33901470}$ $80^{8487270}$ $81^{1052970}$

- A $[96, 9, 71]_{11}$ code:

19A49A4100000000, 5809820983935100, 5799557A58504100, 2957259546935100, 524934122A729200, 698151A07AA41000

0^{1} 71^{2080} 72^{7040} 73^{22880} 74^{71760} 75^{201600} 76^{558080} $77^{1401120}$ $78^{3390160}$ $79^{7730720}$ $80^{16571340}$ $81^{32826560}$ $82^{60072800}$ $83^{101076800}$ $84^{156464680}$ $85^{221215680}$ $86^{282356320}$ $87^{324679040}$ $88^{332088840}$ $89^{298661760}$ $90^{232217920}$ $91^{153303200}$ $92^{83173800}$ $93^{35808320}$ $94^{11398880}$ $95^{2399840}$ 96^{246470}

- A $[114, 9, 87]_{11}$ code:

A4494468373A33901314444A94164100000000, 62872704270737A94112683579A17748074100, 2487301A74101A749A51564381551736394100

0^{1} 87^{3800} 88^{19380} 89^{41420} 90^{130150} 91^{315400} 92^{847400} $93^{1916340}$ $94^{4352140}$ $95^{9075540}$ $96^{18010480}$ $97^{33524360}$ $98^{58269010}$ $99^{94094080}$ $100^{140428050}$ $101^{195163440}$ $102^{249094370}$ $103^{289935820}$ $104^{306678620}$ $105^{292010620}$ $106^{247730360}$ $107^{185732980}$ $108^{120168350}$ $109^{66074780}$ $110^{30056290}$ $111^{10756660}$ $112^{2962290}$ 113^{507300} 114^{48260}

- A $[117, 9, 89]_{11}$ code:

AAAA98708, AAAAA7023, AAAAA9949, AA98113A9, AAA9A9A59, AAA520467, AAA249519, AA9A9A937, AAA721A86, AA9A9A823, AAA9A8973, AA9A89A65, AA9A83687

0^{1} 89^{2430} 90^{9450} 91^{23490} 92^{69660} 93^{186750} 94^{477450} $95^{1162170}$ $96^{2662170}$ $97^{5725080}$ $98^{11707200}$ $99^{22525480}$ $100^{40430070}$ $101^{68276070}$ $102^{106779330}$ $103^{155720790}$ $104^{209514060}$ $105^{259406910}$ $106^{293722830}$ $107^{302114070}$ $108^{279488430}$ $109^{230692770}$ $110^{167979690}$ $111^{106005120}$ $112^{56662830}$ $113^{25050780}$ $114^{8822910}$ $115^{2304180}$ 116^{392040} 117^{33480}

- A $[129, 8, 102]_{11}$ code:

A53022561A5512258769738A5A24250545210000000, 805A4249A6268469160180415631A27892A23498300, 161156A582839A4544946623773255A238816251200

0^{1} 102^{3870} 103^{11180} 104^{29240} 105^{69660} 106^{169850} 107^{365930} 108^{719390} $109^{1385030}$ $110^{2593330}$ $111^{4242380}$ $112^{6922570}$ $113^{10450720}$ $114^{14653540}$ $115^{19243360}$ $116^{23071650}$ $117^{25570380}$ $118^{25993070}$ $119^{24274360}$ $120^{20033270}$ $121^{14921430}$ $122^{9874520}$ $123^{5563770}$ $124^{2684490}$ $125^{1074570}$ 126^{337980} 127^{88580} 128^{10320} 129^{440}

- A $[144, 9, 112]_{11}$ code:

19A49A4100000000, 5809820983935100, 5799557A58504100, 2957259546935100, 524934122A729200, 8743731861A41000, 468083A466140410, 649423A0A2893510, 439A572612120410

0^{1} 112^{2240} 113^{7840} 114^{21440} 115^{59360} 116^{140240} 117^{342880} 118^{763680} $119^{1647680}$ $120^{3460880}$ $121^{6850080}$ $122^{12863440}$ $123^{23057920}$ $124^{39007960}$ $125^{62570560}$ $126^{94344960}$ $127^{134244160}$ $128^{177750860}$ $129^{220066400}$ $130^{254257520}$ $131^{271580160}$ $132^{267114600}$ $133^{241401120}$ $134^{198384400}$ $135^{147031040}$ $136^{97127500}$ $137^{56667040}$ $138^{28764800}$ $139^{12460480}$ $140^{4405440}$ $141^{1250400}$ 142^{262640} 143^{35680} 144^{2290}

□

Theorem 3.5. *There exist 10-dimensional quasi-cyclic codes with parameters:*

$$[30,10,16]_{11},\quad [42,10,26]_{11},\quad [50,10,32]_{11},\quad [60,10,40]_{11},\quad [63,10,42]_{11},$$
$$[76,10,53]_{11},\quad [80,10,56]_{11},\quad [100,10,72]_{11},\quad [110,10,80]_{11},\quad [120,10,89]_{11},$$
$$[130,10,97]_{11},\quad [140,10,106]_{11},\quad [150,10,114]_{11}.$$

Proof. The coefficients of the defining polynomials and weight distributions of the codes are as follows:

- A $[30,10,16]_{11}$ code:

AAAAA90277, AAA99268A9, AAAA6A9A27
0^{1} 16^{2650} 17^{15400} 18^{127550} 19^{815900} $20^{4474820}$ $21^{21322200}$ $22^{86824900}$ $23^{302782500}$ $24^{882327950}$ $25^{2118654360}$ $26^{4073922950}$ $27^{6033437200}$ $28^{6467569700}$ $29^{4458580900}$ $30^{1486565620}$

- A $[42,10,26]_{11}$ code:

A850493290A1000000000, 79980403A860542931010

0^{1} 26^{9870} 27^{47950} 28^{246540} $29^{1205610}$ $30^{5283320}$ $31^{20229300}$ $32^{69520920}$ $33^{211487920}$ $34^{559164690}$ $35^{1277103450}$ $36^{2484728330}$ $37^{4029941370}$ $38^{5299933590}$ $39^{5437603500}$ $40^{4078452210}$ $41^{1988710920}$ $42^{473755110}$

- A $[50,10,32]_{11}$ code:

AAAA228045, AAA9A9A6A0, AAAA616139, AAAA8A6013, AAA999AA15
0^{1} 32^{4350} 33^{25100} 34^{110750} 35^{484820} $36^{2081800}$ $37^{7833600}$ $38^{26855000}$ $39^{82378700}$ $40^{227096640}$ $41^{553829200}$ $42^{1186008000}$ $43^{2206675500}$ $44^{3511313500}$ $45^{4680964760}$ $46^{5089270500}$ $47^{4330191300}$ $48^{2706473150}$ $49^{1104898800}$ $50^{220929130}$

- A $[60,10,40]_{11}$ code:

61353257693A467794841000000000, 9A6879791506839136128433767510
0^{1} 40^{4050} 41^{17700} 42^{81800} 43^{316500} $44^{1270050}$ $45^{4591480}$ $46^{14744700}$ $47^{43916700}$ $48^{119275850}$ $49^{292140900}$ $50^{641974200}$ $51^{1259258800}$ $52^{2179726800}$ $53^{3289704600}$ $54^{4265616500}$ $55^{4651188660}$ $56^{4154147400}$ $57^{2915875200}$ $58^{1507177800}$ $59^{511164300}$ $60^{85230610}$

- A $[63,10,42]_{11}$ code:

A850493290A1000000000, 79980403A860542931010, 1231A2838A421694A1010

0^{1} 42^{2730} 43^{8400} 44^{40110} 45^{162050} 46^{654360} $47^{2315880}$ $48^{7818440}$ $49^{23962260}$ $50^{67072950}$ $51^{170499210}$ $52^{394503270}$ $53^{818401920}$ $54^{1513738590}$ $55^{2478164010}$ $56^{3541994400}$ $57^{4349041900}$ $58^{4498579890}$ $59^{3811614240}$ $60^{2541107310}$ $61^{1250767560}$ $62^{402994620}$ $63^{63980500}$

- A $[76,10,53]_{11}$ code:

A821604991000000000, 94A5950420817560100, 106A83095A6336A9100, 611AA820AA59562A100

0^{1} 53^{3230} 54^{12540} 55^{53960} 56^{200450} 57^{719910} $58^{2313440}$ $59^{7089280}$ $60^{20073500}$ $61^{52628100}$ $62^{127411720}$ $63^{283313560}$ $64^{575047730}$ $65^{1061545580}$ $66^{1769968940}$ $67^{2640571550}$ $68^{3495706450}$ $69^{4053574570}$ $70^{4052512660}$ $71^{3425327600}$ $72^{2378241020}$ $73^{1303210570}$ $74^{528553780}$ $75^{140757890}$ $76^{18586570}$

- A $[80,10,56]_{11}$ code:

AAAA261574, AAAA907686, AAA9A9A726, AAAA5A2726, AAAA656898, AAAA1A2431, AAAA034520, AAAA12A514

0^{1} 56^{1950} 57^{7100} 58^{33200} 59^{126700} 60^{454520} $61^{1480200}$ $62^{4479750}$ $63^{12861300}$ $64^{34163600}$ $65^{83896560}$ $66^{191058800}$ $67^{398979700}$ $68^{763021300}$ $69^{1326735300}$ $70^{2084853270}$ $71^{2936029400}$ $72^{3670819900}$ $73^{4021777800}$ $74^{3805747050}$ $75^{3043809800}$ $76^{2002927450}$ $77^{1040193800}$ $78^{399856650}$ $79^{101416100}$ $80^{12693400}$

- A $[100,10,72]_{11}$ code:

AAAA604869, AAA6566923, AAA9604942, AAA7A2A961, AAA5476469, AAA64A6723, AAA9A05487, AAA8675251, AAA7855165, AAA8788588

0^{1} 72^{600} 73^{2800} 74^{12800} 75^{43800} 76^{154900} 77^{466500} $78^{1373350}$ $79^{3849400}$ $80^{10125120}$ $81^{24898800}$ $82^{57672900}$ $83^{125069800}$ $84^{253266550}$ $85^{477107380}$ $86^{831807650}$ $87^{1338319200}$ $88^{1976766250}$ $89^{2665492700}$ $90^{3258431780}$ $91^{3579746400}$ $92^{3502901150}$ $93^{3013177700}$ $94^{2243792100}$ $95^{1416748740}$ $96^{737903150}$ $97^{304329600}$ $98^{93270900}$ $99^{18797300}$ $100^{1895280}$

- A $[110,10,80]_{11}$ code:

AAA9A870A5, AAA8581217, AAA6015A14, AAAA7A6435, AAA9518A13, AAA4835894, AAAA33A216, AAA7716175, AAA5459196, AAA5445654, AAA8412030

0^{1} 80^{770} 81^{2100} 82^{8100} 83^{27800} 84^{88600} 85^{272700} 86^{761150} $87^{2102700}$ $88^{5533950}$ $89^{13628900}$ $90^{31923090}$ $91^{69986800}$ $92^{144729700}$ $93^{280377400}$ $94^{506337400}$ $95^{853241180}$ $96^{1333354850}$ $97^{1924351900}$ $98^{2551690350}$ $99^{3093481600}$ $100^{3403608450}$ $101^{3369475800}$ $102^{2973460100}$ $103^{2308957800}$ $104^{1553847350}$ $105^{888459780}$ $106^{418968400}$ $107^{156522000}$ $108^{43508450}$ $109^{7986400}$ 110^{729030}

- A $[120,10,89]_{11}$ code:

AAA9A55174, AAA7A53429, AAA5854866, AAA6A15141, AAAA776716, AAA85A5916, AAA548A054, AAA6721071, AAA9514442, AAA9A28109, AAA9248620, AAA6791289

0^{1} 89^{1200} 90^{4020} 91^{14600} 92^{49050} 93^{147200} 94^{434150} $95^{1187620}$ $96^{3050000}$ $97^{7523000}$ $98^{17646400}$ $99^{39247000}$ $100^{82484850}$ $101^{162848900}$ $102^{304192150}$ $103^{531228300}$ $104^{869182150}$ $105^{1322991000}$ $106^{1873371000}$ $107^{2450298200}$ $108^{2949555550}$ $109^{3246747600}$ $110^{3246586500}$ $111^{2926453900}$ $112^{2351008050}$ $113^{1664082600}$ $114^{1021842850}$ $115^{533332080}$ $116^{229769300}$ $117^{78527300}$ $118^{19965450}$ $119^{3372300}$ 120^{280330}

- A $[130, 10, 97]_{11}$ code:

AAA6408413, AAA8415184, AAAA518169, AAAA250235, AAA6393897, AAA5518011, AAA6853235, AAA97229A2, AAA72917A8, AAA8764968, AAAA3972A9, AAA5543219, AAA7206218

0^{1} 97^{700} 98^{2650} 99^{10200} 100^{27940} 101^{87500} 102^{233900} 103^{647800} $104^{1682700}$ $105^{4148500}$ $106^{9837350}$ $107^{21958700}$ $108^{46861300}$ $109^{94474000}$ $110^{180625300}$ $111^{325243000}$ $112^{551924550}$ $113^{878767200}$ $114^{1310707600}$ $115^{1823459300}$ $116^{2358381700}$ $117^{2822018700}$ $118^{3108661200}$ $119^{3134415900}$ $120^{2872671050}$ $121^{2375265000}$ $122^{1752120750}$ $123^{1139788500}$ $124^{643157450}$ $125^{308620980}$ $126^{122409650}$ $127^{38655400}$ $128^{9050150}$ $129^{1398000}$ 130^{109980}

- A $[140, 10, 106]_{11}$ code:

723000842458606A4201186561000000000, 2A08338446793770621582315A689A47710,

64636983266608AA83A7410329470033710, 7159A60A8A974A9A814460A590759608100

0^{1} 106^{2800} 107^{6650} 108^{15400} 109^{52500} 110^{130760} 111^{377300} 112^{935200} $113^{2301600}$ $114^{5490800}$ $115^{12249650}$ $116^{26495350}$ $117^{54628350}$ $118^{106284500}$ $119^{196487550}$ $120^{344300040}$ $121^{568197350}$ $122^{884821000}$ $123^{1297080050}$ $124^{1777577900}$ $125^{2273257000}$ $126^{2706764900}$ $127^{2983994650}$ $128^{3029777800}$ $129^{2818781000}$ $130^{2387099540}$ $131^{1821694350}$ $132^{1241665950}$ $133^{746347700}$ $134^{390349050}$ $135^{173217730}$ $136^{63719950}$ $137^{18609500}$ $138^{4087650}$ 139^{581350} 140^{41730}

- A $[150, 10, 114]_{11}$ code:

AAAA261574, AAA95A4412, AAA9A67368, AAA9A89A89, AAAA5A2726,AAAA656898, AAA9A03703, AAA9897318, AAAA1A2431, AAA9A88436, AAA9871840, AAA9A78023, AAA9A9A897, AAA9A9A726, AAA9861469

0^{1} 114^{1800} 115^{2500} 116^{9650} 117^{29100} 118^{80800} 119^{198900} 120^{528080} $121^{1276100}$ $122^{3045950}$ $123^{6933500}$ $124^{15018650}$ $125^{31355900}$ $126^{62274250}$ $127^{117332000}$ $128^{211438200}$ $129^{360722100}$ $130^{582199420}$ $131^{889035000}$ $132^{1279779200}$ $133^{1731184000}$ $134^{2196639800}$ $135^{2604564060}$ $136^{2870866950}$ $137^{2934584900}$ $138^{2764748750}$ $139^{2386341100}$ $140^{1875015110}$ $141^{1330239800}$ $142^{843018900}$ $143^{471744800}$ $144^{229074500}$ $145^{94775640}$ $146^{32474750}$ $147^{8816700}$ $148^{1827700}$ 149^{229400} 150^{16640}

□

The ratio $R = k/n$ is called rate of the code. A good code will have small n (for fast transmission of messages), large k (to enable transmission of wide variety of messages) and large d (to correct many errors). So for $n \leq 150$ high-rate codes with large d are good codes. We investigate such codes as orthogonal codes to given cyclic or quasi-cyclic codes. Three such codes have been presented in Theorem 3.2. In the following table we present ten more new high-rate codes. The parameters of the codes and the respective orthogonal ones are given. Their weight enumerators are obtained via the MacWilliams' identities.

N:	Code C	Code $C^{\perp}$	N:	Code C	Code $C^{\perp}$
1	$[38, 6, 28]_{11}$	$[38, 32, 5]_{11}$	6	$[70, 8, 52]_{11}$	$[70, 62, 5]_{11}$
2	$[42, 5, 33]_{11}$	$[42, 37, 4]_{11}$	7	$[76, 9, 55]_{11}$	$[76, 67, 6]_{11}$
3	$[42, 7, 25]_{11}$	$[42, 35, 7]_{11}$	8	$[129, 8, 102]_{11}$	$[129, 121, 5]_{11}$
4	$[60, 10, 40]_{11}$	$[60, 50, 6]_{11}$	9	$[133, 6, 111]_{11}$	$[133, 127, 4]_{11}$
5	$[63, 5, 51]_{11}$	$[63, 58, 4]_{11}$	10	$[144, 9, 112]_{11}$	$[144, 135, 5]_{11}$

4. Bounds on $d_{11}(n, k)$

n\k	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
3	1							
4	2	1						
5	3	2	1					
6	4	3	2	1				
7	5	4	3	2	1			
8	6	5	4	3	2	1		
9	7	6	5	4	3	2	1	
10	8	7	6	5	4	3	2	1
11	9	8	7	6	5	4	3	2
12	10	9	8	7	6	5	4	3
13	10	9	8	7	7	5-6	4-5	3-4
14	11	10	9	8	7	6-7	5-6	4-5
15	12	11	10	9	8	7	6-7	5-6
16	13	12	11	**10**	**9**	**8**	7	6-7
17	14	13	11-12	10-11	**10**	**9**	**8**	7
18	15	14	12-13	11-12	10-11	**10**	**9**	8
19	16	15	13-14	12-13	11-12	10-11	**10**	**9**
20	17	16	14-15	13-14	12-13	10-12	10-11	9-10
21	18	16-17	15-16	14-15	13-14	11-13	10-12	9-11
22	18	17-18	16-17	14-16	13-15	12-14	11-13	10-12
23	19	18	17-18	15-17	14-16	13-15	12-14	11-13
24	20	19	18	16-18	15-17	**14**-16	**13**-15	**12**-14
25	21	20	18-19	17-18	15-18	14-17	13-16	12-15
26	22	21	19-20	17-19	16-18	15-18	14-17	12-16
27	23	21-22	20-21	18-20	17-19	**16**-18	**15**-18	13-17
28	24	22-23	21-22	19-21	18-20	16-19	15-18	14-18
29	25	23-24	22-23	20-22	19-21	17-20	16-19	15-18
30	26	24-25	23-24	21-23	20-22	18-21	17-20	**16**-19
31	27	25-26	23-25	22-24	20-23	19-22	18-21	16-20
32	28	26-27	24-26	23-25	21-24	**20**-23	**19**-22	16-21
33	28	27-28	25-27	24-26	22-25	20-24	19-23	17-22
34	29	28	26-28	25-27	23-26	21-25	20-24	18-23
35	30	29	27-28	26-28	24-27	22-26	21-25	19-24
36	31	30	28-29	27-28	25-28	23-27	22-26	20-25
37	32	31	28-30	27-29	26-28	24-28	23-27	21-26
38	33	32	29-31	28-30	27-29	24-28	**24**-28	22-27
39	34	32-33	30-32	29-31	27-30	25-29	24-28	23-28
40	35	33-34	31-33	30-32	28-31	26-30	25-29	24-28

n\k	3	4	5	6	7	8	9	10
41	36	34-35	32-34	30-33	29-32	27-31	26-30	25-29
42	37	35-36	33-35	31-34	30-33	28-32	26-31	**26**-30
43	38	36-37	34-36	32-35	30-34	**29**-33	27-32	26-31
44	38-39	37-38	34-37	33-36	31-35	29-34	28-33	26-32
45	39-40	38-39	35-38	34-37	32-36	30-35	29-34	27-33
46	40	38-40	36-39	35-38	33-37	31-36	30-35	28-34
47	41	39-40	37-40	36-39	34-38	32-37	31-36	29-35
48	42	40-41	38-40	37-40	35-39	**33**-38	**32**-37	30-36
49	43	41-42	39-41	37-40	35-40	33-39	32-38	31-37
50	44	42-43	40-42	38-41	36-40	33-40	32-39	**32**-38
51	45	43-44	40-43	39-42	37-41	34-40	33-40	32-39
52	46	44-45	41-44	40-43	38-42	35-41	34-40	32-40
53	47	44-46	42-45	41-44	39-43	36-42	35-41	33-40
54	48	45-47	43-46	42-45	40-44	37-43	36-42	34-41
55	49	46-48	44-47	43-46	41-45	38-44	37-43	35-42
56	50	47-49	45-48	44-47	42-46	39-45	38-44	36-43
57	50	48-50	46-49	45-48	43-47	40-46	39-45	37-44
58	51	49-50	47-50	45-49	43-48	41-47	40-46	38-45
59	52	50-51	48-50	46-50	44-49	42-48	41-47	39-46
60	53	51-52	49-51	47-50	45-49	43-49	42-48	**40**-47
61	54	52-53	49-52	47-51	45-50	**44**-49	**43**-49	40-48
62	55	52-54	50-53	48-52	46-51	44-50	43-49	41-49
63	56	53-55	51-54	49-53	47-52	45-51	44-50	**42**-49
64	57	54-56	52-55	50-54	48-53	46-52	**45**-51	42-50
65	58	55-57	53-56	51-55	49-54	47-53	45-52	42-51
66	59	56-58	54-57	52-56	50-55	48-54	45-53	43-52
67	60	57-59	55-58	52-57	51-56	49-55	46-54	44-53
68	60	58-60	55-59	53-58	52-57	50-56	47-55	45-54
69	61	59-60	56-60	54-59	53-57	51-57	48-56	46-55
70	62	60-61	57-60	55-60	54-58	**52**-57	49-57	47-56
71	63	61-62	58-61	56-60	55-59	52-58	50-57	48-57
72	64	62-63	59-62	57-61	56-60	**53**-59	**51**-58	49-57
73	65	63-64	60-63	58-62	57-61	53-60	51-59	50-58
74	66	64-65	61-64	59-63	58-62	54-61	52-60	51-59
75	67	65-66	62-65	60-64	58-63	55-62	53-61	52-60
76	68	66-67	62-66	61-65	59-64	56-63	54-62	**53**-61
77	69	67-68	63-67	61-66	59-65	57-64	55-63	53-62
78	70	68-69	64-68	62-67	60-65	58-65	56-64	54-63
79	70	69-70	65-69	63-68	61-66	59-65	57-65	55-64
80	71	70	66-69	64-69	62-67	**60**-66	58-65	**56**-65

n\k	3	4	5	6	7	8	9	10
81	72	70-71	67-70	65-69	62-68	60-67	**59**-66	56-65
82	73	71-72	68-71	66-70	63-69	61-68	59-67	57-66
83	74	72-73	69-72	67-71	64-70	62-69	59-68	58-67
84	75	73-74	70-73	68-72	65-71	63-70	60-69	59-68
85	76	74-75	71-74	68-73	66-72	64-71	61-70	60-69
86	77	75-76	71-75	69-74	67-73	65-72	62-71	61-70
87	78	76-77	72-76	70-75	67-74	66-73	63-72	62-71
88	79	77-78	73-77	71-76	68-74	67-74	64-73	63-72
89	80	78-79	74-78	72-77	69-75	68-74	65-74	64-73
90	80-81	79-80	75-78	73-78	70-76	69-75	66-74	65-74
91	81	80-81	76-79	73-78	71-77	70-76	67-75	66-74
92	82	81	77-80	74-79	72-78	71-77	68-76	67-75
93	83	81-82	78-81	75-80	73-79	72-78	69-77	68-76
94	84	82-83	79-82	76-81	74-80	73-79	70-78	69-77
95	85	83-84	80-83	77-82	75-81	**74**-80	70-79	**70**-78
96	86	84-85	81-84	78-83	76-82	74-81	**71**-80	70-79
97	87	85-86	81-85	79-84	76-83	74-82	71-81	70-80
98	88	86-87	82-86	80-85	77-83	74-83	72-82	70-81
99	89	87-88	83-87	81-86	78-84	75-83	**73**-83	71-82
100	90	88-89	84-87	82-87	79-85	76-84	73-83	**72**-83
101	90-91	89-90	85-88	83-87	79-86	77-85	74-84	72-83
102	91-92	90-91	86-89	84-88	80-87	78-86	75-85	72-84
103	92	91-92	86-90	85-89	81-88	79-87	76-86	73-85
104	93	92	87-91	86-90	82-89	**80**-88	77-87	74-86
105	94	93	88-92	87-91	83-90	80-89	78-88	75-87
106	95	94	89-93	88-92	84-91	81-90	79-89	76-88
107	96	95	90-94	89-93	85-92	82-91	80-90	77-89
108	97	96	91-95	90-94	86-93	83-92	81-91	78-90
109	98	97	92-96	91-95	87-93	84-93	82-92	79-91
110	99	98	93-96	92-96	88-94	85-93	83-93	**80**-92
111	100	99	93-97	93-96	89-95	86-94	84-93	80-93
112	101	100	94-98	93-97	90-96	**87**-95	85-94	81-93
113	102	101	95-99	93-98	91-97	87-96	86-95	82-94
114	103	102	96-100	94-99	92-98	88-97	**87**-96	83-95
115	104	103	97-101	94-100	92-99	89-98	87-97	84-96
116	105	104	98-102	95-101	92-100	90-99	88-98	85-97
117	106	105	99-103	96-102	93-101	91-100	**89**-99	86-98
118	107	106	100-104	97-103	94-102	92-101	89-100	87-99
119	108	107	101-105	98-104	95-102	93-102	89-101	88-100
120	109	108	102-105	99-105	96-103	**94**-102	90-102	**89**-101

n\k	3	4	5	6	7	8	9	10
121	110	109	103-106	99-105	97-104	94-103	91-102	89-102
122	110	110	104-107	100-106	98-105	95-104	92-103	89-102
123	111	110	104-108	101-107	99-106	96-105	93-104	90-103
124	112	110-111	105-109	102-108	100-107	97-106	94-105	91-104
125	113	110-112	106-110	103-109	101-108	98-107	95-106	92-105
126	114	110-113	107-111	104-110	102-109	99-108	**96**-107	93-106
127	115	111-114	108-112	105-111	103-110	100-109	96-108	94-107
128	116	112-115	109-113	106-112	104-111	101-110	97-109	95-108
129	117	113-116	110-114	107-113	105-112	**102**-111	98-110	96-109
130	118	114-117	111-114	108-114	105-112	102-112	99-111	**97**-110
131	119	115-118	111-115	109-114	106-113	102-112	100-112	97-111
132	120	116-119	112-116	110-115	107-114	103-113	101-112	98-112
133	121	117-120	113-117	111-116	108-115	104-114	102-113	99-112
134	121	118-121	114-118	111-117	108-116	105-115	103-114	100-113
135	121	119-121	115-119	112-118	109-117	106-116	**104**-115	101-114
136	122	119-121	116-120	113-119	110-118	**107**-117	104-116	102-115
137	123	120-122	117-121	114-120	110-119	107-118	105-117	103-116
138	124	121-123	118-122	115-121	111-120	108-119	106-118	104-117
139	125	122-124	119-123	116-122	112-121	109-120	107-119	105-118
140	126	123-125	120-124	117-123	113-122	110-121	108-120	**106**-119
141	127	124-126	120-124	118-124	114-122	111-122	109-121	106-120
142	128	125-127	121-125	119-124	115-123	112-122	110-122	106-121
143	129	126-128	122-126	120-125	116-124	113-123	111-122	107-122
144	130	127-129	123-127	121-126	117-125	**114**-124	**112**-123	108-122
145	131	127-130	124-128	122-127	118-126	114-125	112-124	109-123
146	132	128-131	125-129	123-128	119-127	115-126	112-125	110-124
147	132	129-132	126-130	124-129	120-128	116-127	113-126	111-125
148	133	130-132	126-131	125-130	121-129	117-128	114-127	112-126
149	134	131-133	127-132	125-131	122-130	118-129	115-128	113-127
150	135	132-134	128-133	126-132	123-131	119-130	116-129	**114**-128

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